

Animated Logic: Correct Functional Conversion to Conjunctive Normal Form

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Abstract

We present an approach to obtain formally verified implementations of classical Computational Logic algorithms. We choose the Why3 platform because it allows to implement functions in a style very close to the mathematical definitions, as well as it allows a high degree of automation in the verification process.

As proof of concept, we present a mathematical definition of the algorithm to convert propositional formulae to conjunctive normal form, implementations in WhyML (the Why3 language, very similar to OCaml), and proofs of correctness of the implementations. We apply our proposal on two variants of this algorithm: one in direct-style and another with an explicit stack structure. Being both first-order versions, Why3 processes the proofs naturally.

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1 Introduction

Motivation. Foundational courses in Computer Science, like Computational Logic, aim at presenting key basilar subjects to the education of undergraduate students. To strength the relation of the topics covered to sound programming practices, it is relevant to link the mathematical content to clear and executable implementations, provably correct to stress the importance of sound practices.

Herein we present work developed in the context of the FACTOR [2] project, which aims to promote the use of OCaml [10] and correct code development practices in the Portuguese-speaking academic community. Specifically, the objectives of the project are the functional implementation of classical Computational Logic algorithms and Formal Languages, the accomplishment of correctness proofs and the step-by-step execution to help understanding the algorithm.

The algorithm for converting propositional formulae to Conjunctive Normal Form (CNF)¹ is often presented formally, with rigorous mathematical definitions that are sometimes difficult to read [5, 7, 11], or informally, intended for Computer Science but with textual definitions in non-executable pseudo-code [4, 9]. The implementation of algorithms of this nature is a fundamental piece for learning and understanding them. Languages such OCaml allow implementations very close to the mathematical definitions, helping the study because they are executable. Also, the correctness proof of a functional implementation is simpler than for the imperative one.

¹ A formula is in CNF if it is a conjunction of clauses, where a clause is a disjunction of literals and a literal is a propositional symbol or its negation.

Contributions. As proof of concept, we implement and prove correct the referred algorithm in Why3 [6], a platform for deductive program verification. Why3 provides a first-order language with polymorphic types, pattern matching and inductive predicates, called WhyML. Also offers a certified OCaml code extraction mechanism and support for third-party provers.

To support the step-by-step execution of the algorithm, an important feature to help students understanding the definitions, we also implemented a version in Continuation-Passing Style (CPS) [14] and via defunctionalization got an evaluator, a version close to a first-order abstract machine [3]. Due to the limited support of Why3 to the higher order, it was not possible to close the correctness proof for the CPS version. This limitation however is not present in the defunctioned implementation that has an explicit stack structure, but in first-order. This implementation resulted from a mechanical transformation from the CPS version. This version has been naturally proven correct by Why3.

In short, this article presents pedagogical material to support the teaching of classical Computational Logic algorithms. We developed two implementations, formally verified in Why3, from a presentation as a recursive function of conversion algorithm to CNF: the first in direct style and the second with an explicit stack structure. Both were proved sound with small effort, basically following from the assertions one naturally associates with the code to prove it correct.

2 Functional presentation of the algorithm

Description. Let us call T to the algorithm that converts any propositional logic formula to CNF. A propositional formula ϕ is an element of the set G_p , defined as follows:

$$G_p \triangleq \phi ::= T \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi$$

The function T produces formula without the implication connective, so we define also the set H_p as a subset of G_p without implications. We thus have $T: G_p \rightarrow H_p$, where:

$$T(\phi) = \text{CNFC}(\text{NNFC}(\text{Impl_Free}(\phi)))$$

The algorithm composes three functions:

- The `Impl_Free` function responsible for eliminating the implications;
- The `NNFC` function responsible for converting to Negation Normal Form (NNF)²
- The `CNFC` function responsible for converting from NNF to CNF.

Implementation. To represent the set G_p , we define the type `formula` that declares variables (`FVar`) and constants (`FConst`), and constructs formulas with conjunctions (`FAnd`), disjunctions (`FOr`), implications (`FImpl`) and negations (`FNeg`):

```
type formula =
  | FVar ident
  | FConst bool
  | FAnd formula formula
  | FOr formula formula
  | FImpl formula formula
  | FNeg formula
```

² A formula is in NNF if the negation operator is only applied to sub-formulae that are literals.

To represent the H_p set we define the type `formula_wi`, similar to the previous but without the implication constructor.

We present now the implementation of the three functions.

The function `Impl_Free` removes all the implications. It is recursively defined in the cases of the type `formula` and homomorphic, except in the implication case where it takes advantage of the Propositional Logic Law:

$$A \rightarrow B \equiv \neg A \vee B$$

It converts the constructions of the type `formula` for those of the type `formula_wi` and does recursive calls over the arguments:

```
let rec impl_free (phi: formula) : formula_wi
= match phi with
| FNeg phi1 → FNeg_wi (impl_free phi1)
| FOr phi1 phi2 → FOr_wi (impl_free phi1) (impl_free phi2)
| FAnd phi1 phi2 → FAnd_wi (impl_free phi1) (impl_free phi2)
| FImpl phi1 phi2 → FOr_wi (FNeg_wi (impl_free phi1)) (impl_free phi2)
| FConst phi → FConst_wi phi
| FVar phi → FVar_wi phi
end
```

The functions `NNFC` converts formulas to NNF. It is recursively defined over a combination of constructors: applying the Propositional Logic Law $\neg\neg A \equiv A$ the double negations are eliminated and using the De Morgan Laws, negations of conjunctions become disjunction of negations and negations of disjunctions become conjunction of negations. The code of the function is as follows:

```
let rec nnfc (phi: formula_wi) : formula_wi
= match phi with
| FNeg_wi (FNeg_wi phi1) → nnfc phi1
| FNeg_wi (FAnd_wi phi1 phi2) → FOr_wi (nnfc (FNeg_wi phi1))
  (nnfc (FNeg_wi phi2))
| FNeg_wi (FOr_wi phi1 phi2) → FAnd_wi (nnfc (FNeg_wi phi1))
  (nnfc (FNeg_wi phi2))
| FOr_wi phi1 phi2 → FOr_wi (nnfc phi1) (nnfc phi2)
| FAnd_wi phi1 phi2 → FAnd_wi (nnfc phi1) (nnfc phi2)
| phi → phi
end
```

The `CNFC` function converts formulas from NNF to CNF. It is straightforwardly defined except in the disjunction case, where it distributes the disjunction by the conjunction calling the auxiliary function `distr`.

```
let rec cnfc (phi: formula_wi) : formula_wi
= match phi with
| FOr_wi phi1 phi2 → distr (cnfc phi1) (cnfc phi2)
| FAnd_wi phi1 phi2 → FAnd_wi (cnfc phi1) (cnfc phi2)
| phi → phi
end
```

The `distr` function uses the Propositional Logic Law

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C),$$

the code is the following:

```

let rec distr (phi1 phi2: formula_wi) : formula_wi
= match phi1, phi2 with
| FAnd_wi phi11 phi12, phi2 → FAnd_wi (distr phi11 phi2)
  (distr phi12 phi2)
| phi1, FAnd_wi phi21 phi22 → FAnd_wi (distr phi1 phi21)
  (distr phi1 phi22)
| phi1, phi2 → FOr_wi phi1 phi2
end

```

Lastly, the code of the function (T) composes all of these functions:

```

let t (phi: formula) : formula_wi
= cnfc(nnfc(impl_free phi))

```

From this WhyML implementation it is possible to extract OCaml code (Appendix A). Both implementations are close to the mathematical definitions thus demonstrating that functional languages like OCaml are suitable languages for the presentation of these algorithms, providing executable definitions without sacrificing rigor or clarity.

3 How to obtain the correctness

Since the T algorithm is a composition of three functions, the correctness of the algorithm is the result of the correctness criteria of each of these three functions.

Criteria. For all functions the basic correctness criterion is that the input and output formula must be logically equivalent. In addition it is required that:

- **Impl_Free:**
 - The result should not contain implications.
- **NNFC:**
 - The input and output formula should not contain implications.
 - The result must be in Negation Normal Form.
- **CNFC:**
 - The input and output formula should not contain implications.
 - The input and output formula must be in Negation Normal Form.
 - The result must be in Conjunctive Normal Form.

The correctness criteria of a function needs to be propagated to the following ones, to ensure that each does not violate the conditions already established by the one previously executed.

Semantics of formulae. Since the basic criterion of correctness is the logical equivalence of formulae, we need a function to assign a semantic to them:

```

type valuation = ident → bool

function eval (v: valuation) (f: formula) : bool
= match f with
| FVar x      → v x
| FConst b    → b
| FAnd f1 f2  → eval v f1 ∧ eval v f2
| FOr f1 f2   → eval v f1 || eval v f2
| FImpl f1 f2 → (eval v f1 → eval v f2)
| FNeg f      → not (eval v f)
end

```

This function takes an argument of type `valuation` assigning a value of type `bool`³ to each variable of the formula, receives the formula to evaluate and returns a value of type `bool`. For the base constructors `FVar` and `FConst`, the Boolean value of the variable and the value of the constant, respectively, are returned. For the remaining constructor cases, the associated formulae are recursively evaluated and the result translated into the corresponding WhyML Boolean operation. The valuation function for the type of formulae `formula_wi` is similar.

4 Proof of correctness

The proof of correctness consists in demonstrating that each function respects the correctness criteria defined in the previous section. We show herein the WhyML code accepted by Why3 as correct.

Correctness of Impl_Free. The absence of implication connectives is ensured by the return type of the function (`formula_wi`); the equivalence of the formulae is ensured using the formula valuation functions and we use the input formula as a measure to ensure termination.

```
let rec function impl_free (phi: formula) : formula_wi
  ensures { forall v. eval v phi = eval_wi v result }
  variant { phi }
= ...
```

Correctness of NNFC. The absence of implication connectives in the input and output formulae is ensured by the type `formula_wi`. Additionally, to prove that the result is in the NNF, we introduce a well-formedness predicate `wf_negations_of_literals`, which states that the sub-formulae of the constructor `FNeg_wi` cannot contain the constructors `FOr_wi`, `FAnd_wi`, or `FNeg_wi`:

```
predicate wf_negations_of_literals (f: formula_wi)
= match f with
| FNeg_wi f → (forall f1 f2. f ≠ FOr_wi f1 f2 ∧ f ≠ FAnd_wi f1 f2 ∧ f ≠
  FNeg_wi f1) ∧ wf_negations_of_literals f
| FOr_wi f1 f2 | FAnd_wi f1 f2 → wf_negations_of_literals f1 ∧
  wf_negations_of_literals f2
| FVar_wi _ → true
| FConst_wi _ → true
end
```

In the proof of correctness it is not possible to use the formula itself as a measure of termination, since in the case of the distribution of negation by conjunction or disjunction, constructors are added to the head constructors, making the structural inductive criterion not applicable. Hence, we define a function that counts the number of constructors of each formula and use it as termination measure:

```
function size (phi: formula_wi) : int
= match phi with
| FVar_wi _ | FConst_wi _ → 1
| FNeg_wi phi → 1 + size phi
| FAnd_wi phi1 phi2 | FOr_wi phi1 phi2 → 1 + size phi1 + size phi2
end
```

³ `bool` is the Boolean type of WhyML.

To ensure the number of constructors can never be negative, we use an auxiliary lemma:

```
let rec lemma size_nonneg (phi: formula_wi)
  variant { phi }
  ensures { size phi ≥ 0 }
= match phi with
| FVar_wi _ | FConst_wi _ → ()
| FNeg_wi phi → size_nonneg phi
| FAnd_wi phi1 phi2 | FOr_wi phi1 phi2 →
  size_nonneg phi1; size_nonneg phi2
end
```

So, now with the well-formedness predicate and termination measure defined, we can close the proof of correctness of the NNFC function:

```
let rec nnfc (phi: formula_wi) : formula_wi
  ensures { forall v. eval_wi v phi = eval_wi v result }
  ensures { wf_negations_of_literals result }
  variant { size phi }
= ...
```

Correctness of CNFC. To ensure that a given formula is in CNF we introduce the well-formedness predicates `wf_conjunctions_of_disjunctions` and `wf_disjunctions`. These guarantee that after a disjunction there are no conjunctions:

```
predicate wf_conjunctions_of_disjunctions (f: formula_wi)
= match f with
| FAnd_wi f1 f2 → wf_conjunctions_of_disjunctions f1 ∧
  wf_conjunctions_of_disjunctions f2
| FOr_wi f1 f2 → wf_disjunctions f1 ∧ wf_disjunctions f2
| FConst_wi _ → true
| FVar_wi _ → true
| FNeg_wi f1 → wf_conjunctions_of_disjunctions f1
end

predicate wf_disjunctions (f: formula_wi)
= match f with
| FAnd_wi _ _ → false
| FOr_wi f1 f2 → wf_disjunctions f1 ∧ wf_disjunctions f2
| FConst_wi _ → true
| FVar_wi _ → true
| FNeg_wi f1 → wf_disjunctions f1
end
```

Lastly, we add the predicates `wf_conjunctions_of_disjunctions` and `wf_negations_of_literals` to the post-conditions, to ensure that the result is in NNF and CNF, respectively; to ensure that the input formula is in NNF, we also add the predicate `wf_negations_of_literals` to the pre-conditions. The code is as follows:

```
let rec cnfc (phi: formula_wi)
  requires { wf_negations_of_literals phi }
  ensures { forall v. eval_wi v phi = eval_wi v result }
  ensures { wf_negations_of_literals result }
  ensures { wf_conjunctions_of_disjunctions result }
  variant { phi }
= ...
```

Since the CNFC function uses the auxiliary function `distr`, we also need to prove its correctness. We need to ensure the same criteria of the CNFC function, but because it is the distribution of the disjunctions by the conjunctions, we must additionally ensure that the input formulae are in the CNF, which is obtained by adding the predicates `wf_conjunctions_of_disjunctions` and `wf_negations_of_literals` to the pre-conditions:

```
let rec distr (phi1 phi2: formula_wi)
  requires { wf_negations_of_literals phi1 }
  requires { wf_negations_of_literals phi2 }
  requires { wf_conjunctions_of_disjunctions phi1 }
  requires { wf_conjunctions_of_disjunctions phi2 }
  ensures { forall v. eval_wi v (FOr_wi phi1 phi2) = eval_wi v result }
  ensures { wf_negations_of_literals result }
  ensures { wf_conjunctions_of_disjunctions result }
  variant { size phi1 + size phi2 }
= ...
```

However in the `distr` function, it is not possible to prove that a disjunction of two formulae in CNF is effectively a formula in CNF, because we must ensure that in a disjunction of two formulae in CNF, the formulae do not contain the constructor `FAnd_wi`. To accomplish this, we use an auxiliary lemma:

```
lemma aux: forall x. wf_conjunctions_of_disjunctions x →
  wf_negations_of_literals x → not (exists f1 f2. x = FAnd_wi f1 f2) →
  wf_disjunctions x
```

Correctness of T. With the proofs of correctness of each of the three functions performed, we can now obtain the proof of correctness of the function T. This guarantees the criteria ensured by the three functions: it does not contain implications due to the return type (`formula_wi`); is in NNF (line 2); is in CNF (line 3); the result is equivalent to the input formula (line 4):

```
let t (phi: formula) : formula_wi
  ensures { wf_negations_of_literals result }
  ensures { wf_conjunctions_of_disjunctions result }
  ensures { forall v. eval v phi = eval_wi v result }
= cnfc (nmfc (impl_free phi))
```

The proof of this "direct style" implementation specification - close to classical mathematical definitions - is immediate in Why3, making this exercise a successful proof of concept: classical logical algorithms presented as functions to undergraduates can have a very close functional implementation that is easy to prove correct with a high degree of automation.

5 Continuation-Passing Style

Continuation-Passing Style (CPS) is a programming style where the control is passed explicitly in the form of a continuation, thus avoiding the overflow of the stack if the underlying compiler optimizes recursive terminal calls. With an explicit stack structure in the code, it is possible in the future to introduce a mechanism that allows step-by-step execution of the functions.

Process transformation into CPS. The transformation is performed mechanically according to the following steps:

- Given a function of type $\tau' \rightarrow \tau$, we add an argument which represents the continuation (a function of type $\tau \rightarrow 'a$) and change the return type of the function to $'a$.
- For the base cases instead of returning the desired values, we apply these values to the continuation function.
- For the remaining cases, we start by making a recursive call and construct the continuations with the rest of the computation.
- We add a `main` function that calls the function in CPS with the identity function as a continuation.

We apply now this process to the functions presented in the previous section. With respect to the `Impl_Free` function:

1. We add an argument to the function of type `formula_wi` $\rightarrow 'a$ and change the return type to $'a$:

```
let rec impl_free_cps (phi: formula) (k: formula_wi → 'a) : 'a
```

2. For the base cases, we apply the desired values to the continuation function.

```
| FConst phi → k (FConst_wi phi)
| FVar phi → k (FVar_wi phi)
```

3. For the remaining cases, we start with a recursive call and define the continuations with the rest of the computation:

```
| FOr phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun
con1 → k (FOr_wi con con1)))
| FAnd phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun
con1 → k (FAnd_wi con con1)))
| FImpl phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun
con1 → k (FOr_wi (FNeg_wi con) con1)))
```

4. Finally, we create the `main` function that calls the function in CPS with the identity function as a continuation:

```
let impl_free_main (phi: formula) : formula_wi
= impl_free_cps phi (fun x → x)
```

We obtain the CPS version of the remaining functions in a similar way to this process (the complete code is in Appendix B).

Correctness criteria. One interesting aspect of the proof of correctness of the functions in CPS is the use of the corresponding function in direct style, since these are pure and total functions, as specification, *i.e.*, we simply assure that the result is equal to the result of the functions in direct style, applied to the continuation.

For the `Impl_Free` function in CPS, it is enough to ensure that the result is equal to the result of the direct-style `Impl_Free` function applied to the continuation:

```
let rec impl_free_cps (phi: formula) (k: formula_wi → 'a) : 'a
  variant { phi }
  ensures { result = k(impl_free phi) }
= ...
```

The specification of the function in direct style is then also applied to the function `main`, responsible for calling the CPS functions with the identity function as a continuation:

```
let impl_free_main (phi: formula) : formula_wi
  ensures { forall v. eval v phi = eval_wi v result }
= ...
```

The specifications of the NNFC and CNFC functions in CPS are similar to the specification of the `Impl_Free` function referred above (Appendix C). However for the CNFC function it is necessary to prove its pre-conditions. In particular, it is necessary to prove that the input formula is in NNF.

A proof obligation is generated regarding the validity of the pre-condition whenever a recursive call is made within a continuation. In order to prove such a proof obligation, we need to specify the nature of the continuation arguments. Thus, we encapsulate the `wf_negations_of_literals` predicate into a new type (an invariant type):

```
type nnfc_type = {
  nnfc_formula : formula_wi
} invariant { wf_negations_of_literals nnfc_formula }
by{ nnfc_formula = FConst_wi true }
```

Since the return type of the function is changed, the proof of the post-conditions now involves the comparison of two invariant types, which raises some interesting challenges.

Difficulties preventing a proof. Comparing two invariant types involves providing them a witness, *i.e.*, values with the concerned type; only then it is possible to prove that two values of the same type respect the invariant. However as the invariant type in Why3 is an opaque type, having only access to its projections, it is not possible to construct an inhabitant of this type in the logic, thus making it impossible to compare them. This lemma translates such a behavior:

```
lemma types: forall x y. x.cnfc_formula = y.cnfc_formula → x = y
```

It is not possible to prove this lemma because having only access to record projections can not ensure that, in this case, the field `cnfc_formula` is the only field of this *record* type.

Given this limitation of Why3 [1], which in this case precludes the proof of the post-condition, we have tried to compare the formula of each type with an extensional equality predicate (`==`) and use this predicate as post-condition instead of polymorphic structural equality (`=`).

```
predicate (==) (t1 t2: cnfc_type) = t1.cnfc_formula = t2.cnfc_formula
```

Even with extensional equality, it was not possible to complete the proof. This is due to the fact that for the base cases, given the application to the continuation, we always come across with comparison of records and in the other cases it is not possible to specify the functions of continuation in the recursive calls. This lack of success led to the search for other approaches that would, eventually, achieve the same advantages as the CPS transformation.

What is the problem with CPS? The transformation in CPS always adds a function as an argument, thus passing to a higher order function. Since Why3 is a platform that, for reasons of decidability, operates on a first-order language, the solution is to "go back" to first order. The defunctionalization technique emerged as a possible approach.

6 Defunctionalization

Defunctionalization is a program transformation technique to convert high-order programs into first-order ones [13].

Transformation process. A defunctionalization consists of a "mechanical" transformation in two steps:

1. Get a first order representation of the function continuations and replace the continuations with this new representation.
2. Generate a new a function `apply` which replaces the applications of functions in the original program.

Applying this process to the `Impl_Free` function in CPS lead us to represent the function continuations in first-order:

```
type impl_kont =
  | KImpl_Id
  | KImpl_Neg impl_kont formula
  | KImpl_OrLeft formula impl_kont
  | KImpl_OrRight impl_kont formula_wi
  | KImpl_AndLeft formula impl_kont
  | KImpl_AndRight impl_kont formula_wi
  | KImpl_ImplLeft formula impl_kont
  | KImpl_ImplRight impl_kont formula_wi
```

The constructor `KImpl_id` represents the identity function, the constructor `KImpl_Neg` represents the continuation of the case of the constructor `FNeg_wi`. As the remaining cases contain two continuation functions, two constructors are created, one `left` and one `right`. We chose to use the `left` and `right` nomenclatures because this represents the natural order of the formula in the abstract syntax tree.

We then replace the continuations with the new representation of the continuations:

```
let rec impl_free_desf_cps (phi: formula) (k: impl_kont) : formula_wi
= match phi with
  | FNeg phi1 → impl_free_desf_cps phi1 (KImpl_Neg k phi1)
  | FOr phi1 phi2 → impl_free_desf_cps phi1 (KImpl_OrLeft phi2 k)
  | FAnd phi1 phi2 → impl_free_desf_cps phi1 (KImpl_AndLeft phi2 k)
  | FImpl phi1 phi2 → impl_free_desf_cps phi1 (KImpl_ImplLeft phi2 k)
  ...
end
```

The next step is to introduce an `apply` function, and replace the applications to the continuation:

```
with impl_apply (phi: formula_wi) (k: impl_kont) : formula_wi
= match k with
  | KImpl_Id → phi
  | KImpl_Neg k phi1 → impl_apply (FNeg_wi phi) k
  | KImpl_OrLeft phi1 k → impl_free_desf_cps phi1 (KImpl_OrRight k phi)
  | KImpl_OrRight k phi2 → impl_apply (FOr_wi phi2 phi) k
  | KImpl_AndLeft p k → impl_free_desf_cps p (KImpl_AndRight k phi)
  | KImpl_AndRight k phi2 → impl_apply (FAnd_wi phi2 phi) k
  | KImpl_ImplLeft phi1 k → impl_free_desf_cps phi1 (KImpl_ImplRight k phi)
  | KImpl_ImplRight k phi2 → impl_apply (FOr_wi (FNeg_wi phi2) phi) k
end
```

```

let rec impl_free_desf_cps (phi: formula) (k: impl_kont) : formula_wi
= match phi with
...
| FConst phi → impl_apply (FConst_wi phi) k
| FVar phi → impl_apply (FVar_wi phi) k
end

```

The result of the application of the defunctionalization transformation to the remaining functions of the T algorithm in CPS is in Appendix D.

Proof of correctness. The defunctionalized program specification is the same as the original program. However, given the existence of an additional function generated by the defunctionalization process (the `apply` function), a specification must be provided. Since the `apply` function simulates the application of a function to its argument, the only specification we can give it is that its post-condition is the post-condition of the function `k` [12].

To be able to use the direct-style functions as a specification, we have created a `post` predicate that gathers the post-conditions of the direct-style function. As for the `apply` function, such a predicate performs case analysis on the continuation type and for each constructor, we copy the post-condition present in the corresponding abstraction [12]. For instance, for the `Impl_Free` function, we provide the following specification

```

let rec impl_free_desf_cps (phi: formula) (k: impl_kont) : formula_wi
ensures { impl_post k (impl_free phi) result }
= ...

with impl_apply (phi: formula_wi) (k: impl_kont) : formula_wi
ensures { impl_post k phi result }
= ...

```

where the `impl_post` predicate is:

```

predicate impl_post (k: impl_kont) (phi result: formula_wi)
= match k with
| KImpl_Id → let x = phi in x = result
| KImpl_Neg k phi1 → let neg = phi in impl_post k (FNeg_wi phi) result
| KImpl_OrLeft phi1 k → let hl = phi in impl_post k (FOr_wi phi
  (impl_free phi1)) result
| KImpl_OrRight k phi2 → let hr = phi in impl_post k (FOr_wi phi2 hr) result
| KImpl_AndLeft phi1 k → let hl = phi in impl_post k (FAnd_wi phi
  (impl_free phi1)) result
| KImpl_AndRight k phi2 → let hr = phi in impl_post k
  (FAnd_wi phi2 hr) result
| KImpl_ImplLeft phi1 k → let hl = phi in impl_post k (FOr_wi
  (FNeg_wi phi) (impl_free phi1)) result
| KImpl_ImplRight k phi2 → let hr = phi in impl_post k (FOr_wi
  (FNeg_wi phi2) hr) result
end

```

The proof of the post-conditions of the NNFC and CNFC defunctionalized functions is similar to the proof of the `Impl_Free` function (Appendix E). However, similar to the CPS proof, for the CNFC function, we have to prove its pre-conditions. For this we have created the invariant type `wf_cnfc_kont` with the well-formulated predicate `wf_cnfc_kont` as invariant:

```

type wf_cnfc_kont = {

```

```

    cnfc_k: cnfc_kont;
  } invariant { wf_cnfc_kont cnfc_k }
  by { cnfc_k = KCnfc_Id }

```

Note that in this well-formulated predicate we just want to ensure the CNF for the formulae that are already converted. Given that the formulae are only converted in the right continuation, these and only these feature the `wf_conjunctions_of_disjunctions` predicate:

```

predicate wf_cnfc_kont (phi: cnfc_kont)
= match phi with
| KCnfc_Id → true
| KCnfc_OrLeft phi k → wf_negations_of_literals phi ∧ wf_cnfc_kont k
| KCnfc_OrRight k phi → wf_negations_of_literals phi ∧
  wf_conjunctions_of_disjunctions phi ∧ wf_cnfc_kont k
| KCnfc_AndLeft phi k → wf_negations_of_literals phi ∧ wf_cnfc_kont k
| KCnfc_AndRight k phi → wf_negations_of_literals phi ∧
  wf_conjunctions_of_disjunctions phi ∧ wf_cnfc_kont k
end

```

Lastly, the proof of the T function turns out to be similar to the direct-style proof referenced in Page 7:

```

let t (phi: formula) : formula_wi
  ensures { forall v. eval v phi = eval_wi v result }
  ensures { wf_negations_of_literals result }
  ensures { wf_conjunctions_of_disjunctions result }
= cnfc_desf_main(nnfc_desf_main(impl_desf_main phi))

```

Results. The proof of correctness of the defunctionalized version of the T algorithm is naturally processed by Why3, with each proof objective being proved in less than one second as shown in Figure 1.

7 Conclusion

Functional languages such as OCaml allow close implementations of mathematical definitions without sacrificing clarity and rigor. These make them adequate to be pedagogically used as an aid to the study and understanding of algorithms.

In this article, we present a proof of concept: the implementation and correctness proof of the algorithm to convert propositional formulae to the Conjunctive Normal Form. Proof of the two strands of the algorithm - direct style and defunctionalized - were achieved naturally by Why3, making successful the proof of concept of formally verified implementations of Computational Logic algorithms.

We intend to implement the step-by-step execution, through an explicit stack structure, since each function call returns a function (continuation) that can be used as a block, thus allowing to stop and return the execution. We also intend to apply this approach to other Computational Logic algorithms, such as the Horn [8] algorithm.

Proof obligations		Alt-Ergo 2.3.0	CVC4 1.5	Z3 4.5.0
lemma VC for wf_distr_kont			0.07	
lemma VC for wf_cnfc_kont			0.07	
lemma VC for var_kont_k_nonneg				0.06
lemma VC for impl_free_desf_cps			0.22	
lemma VC for impl_apply			0.27	
lemma VC for impl_desf_main			0.13	
lemma VC for nnfc_desf_cps		0.12		
lemma VC for nnfc_apply			0.53	
lemma VC for nnfc_desf_main			0.12	
lemma VC for distr_desf_cps	lemma precondition		0.17	
	lemma precondition		0.21	
	lemma precondition		0.15	
	lemma postcondition	0.19		
	lemma precondition		0.17	
	lemma precondition		0.13	
	lemma precondition		0.13	
	lemma postcondition	0.43		
	lemma precondition		0.16	
	lemma precondition		0.15	
	lemma precondition		0.14	
	lemma postcondition	0.36		
	lemma precondition		0.11	
	lemma precondition		0.17	
	lemma postcondition	0.81		
lemma VC for distr_apply			0.20	
lemma VC for distr_desf_main			0.12	
lemma VC for cnfc_desf_cps		0.17		
lemma VC for cnfc_apply		0.45		
lemma VC for cnfc_desf_main			0.11	
lemma VC for t			0.14	

■ **Figure 1** Statistical result of defunctionalization proof obligations

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A OCaml code extracted from the WhyML implementation

```

type ident = string

type formula =
  | FVar of ident
  | FConst of bool
  | FAnd of formula * formula
  | FOr of formula * formula
  | FImpl of formula * formula
  | FNeg of formula

type formula_wi =
  | FVar_wi of ident
  | FConst_wi of bool
  | FAnd_wi of formula_wi * formula_wi
  | FOr_wi of formula_wi * formula_wi
  | FNeg_wi of formula_wi

let rec impl_free (phi: formula) : formula_wi =
  begin match phi with
  | FNeg phi1 → FNeg_wi (impl_free phi1)
  | FOr (phi1, phi2) → FOr_wi ((impl_free phi1), (impl_free phi2))
  | FAnd (phi1, phi2) → FAnd_wi ((impl_free phi1), (impl_free phi2))
  | FImpl (phi1, phi2) → FOr_wi ((FNeg_wi (impl_free phi1)), (impl_free phi2))
  | FConst phi1 → FConst_wi phi1
  | FVar phi1 → FVar_wi phi1
  end

let rec nnfc (phi: formula_wi) : formula_wi =
  begin match phi with
  | FNeg_wi (FNeg_wi phi1) → nnfc phi1
  | FNeg_wi (FAnd_wi (phi1, phi2)) → FOr_wi ((nnfc (FNeg_wi phi1)), (nnfc (FNeg_wi phi2)))
  | FNeg_wi (FOr_wi (phi1, phi2)) → FAnd_wi ((nnfc (FNeg_wi phi1)), (nnfc (FNeg_wi phi2)))
  | FOr_wi (phi1, phi2) → FOr_wi ((nnfc phi1), (nnfc phi2))
  | FAnd_wi (phi1, phi2) → FAnd_wi ((nnfc phi1), (nnfc phi2))
  | phi1 → phi1
  end

let rec distr (phi1: formula_wi) (phi2: formula_wi) : formula_wi =
  begin match (phi1, phi2) with
  | (FAnd_wi (phi11, phi12), phi21) →
    let o = distr phi12 phi21 in let o1 = distr phi11 phi21 in FAnd_wi (o1, o)
  | (phi11, FAnd_wi (phi21, phi22)) →
    let o = distr phi11 phi22 in let o1 = distr phi11 phi21 in FAnd_wi (o1, o)
  | (phi11, phi21) → FOr_wi (phi11, phi21)
  end

let rec cnfc (phi: formula_wi) : formula_wi =
  begin match phi with
  | FOr_wi (phi1, phi2) → let o = cnfc phi2 in let o1 = cnfc phi1 in distr o1 o
  | FAnd_wi (phi1, phi2) →

```

```
    let o = cnfc phi2 in let o1 = cnfc phi1 in FAnd_wi (o1, o)
  | phi1 → phi1
end

let t (phi: formula) : formula_wi = cnfc (nnfc (impl_free phi))
```

B CPS Version

```

let rec impl_free_cps (phi: formula) (k: formula_wi → 'a) : 'a
= match phi with
| FNeg phi1 → impl_free_cps phi1 (fun con → k (FNeg_wi con))
| FOr phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun con1 →
  k (FOr_wi con con1)))
| FAnd phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun con1
  → k (FAnd_wi con con1)))
| FImpl phi1 phi2 → impl_free_cps phi1 (fun con → impl_free_cps phi2 (fun con1
  → k (FOr_wi (FNeg_wi con) con1)))
| FConst phi → k (FConst_wi phi)
| FVar phi → k (FVar_wi phi)
end

let impl_free_main (phi: formula) : formula_wi
= impl_free_cps phi (fun x → x)

let rec nnfc_cps (phi: formula_wi) (k: formula_wi → 'a) : 'a
= match phi with
| FNeg_wi phi1 → nnfc_cps phi1 (fun con → k con)
| FNeg_wi (FAnd_wi phi1 phi2) → nnfc_cps (FNeg_wi phi1) (fun con → nnfc_cps (
  FNeg_wi phi2) (fun con1 → k (FOr_wi con con1)))
| FNeg_wi (FOr_wi phi1 phi2) → nnfc_cps (FNeg_wi phi1) (fun con → nnfc_cps (
  FNeg_wi phi2) (fun con1 → k (FAnd_wi con con1)))
| FOr_wi phi1 phi2 → nnfc_cps phi1 (fun con → nnfc_cps phi2 (fun con1 → k (
  FOr_wi con con1)))
| FAnd_wi phi1 phi2 → nnfc_cps phi1 (fun con → nnfc_cps phi2 (fun con1 → k (
  FAnd_wi con con1)))
| phi → k (phi)
end

let nnfc_main (phi: formula_wi) : formula_wi
= nnfc_cps phi (fun x → x)

let rec distr_cps (phi1 phi2: formula_wi) (k: formula_wi → 'a) : 'a
= match phi1, phi2 with
| FAnd_wi phi11 phi12, phi2 → distr_cps phi11 phi2 (fun con → distr_cps phi12
  phi2 (fun con1 → k (FAnd_wi con con1)))
| phi1, FAnd_wi phi21 phi22 → distr_cps phi1 phi21 (fun con → distr_cps phi1
  phi22 (fun con1 → k (FAnd_wi con con1)))
| phi1, phi2 → k (FOr_wi phi1 phi2)
end

let distr_main (phi1 phi2: formula_wi) : formula_wi
= distr_cps phi1 phi2 (fun x → x)

let rec cnfc_cps (phi: formula_wi) (k: formula_wi → 'a) : 'a
= match phi with
| FOr_wi phi1 phi2 → cnfc_cps phi1 (fun con → cnfc_cps phi2 (fun con1 →
  distr_cps con con1 k))
| FAnd_wi phi1 phi2 → cnfc_cps phi1 (fun con → cnfc_cps phi2 (fun con1 → k (
  FAnd_wi con con1)))
| phi → k (phi)

```

```
end  
  
let cnfc_main (phi: formula_wi) : formula_wi  
= cnfc_cps phi (fun x → x)
```

C Correction of functions in CPS

C.1 NNFC

```
let rec nnfc_cps (phi: formula_wi) (k: formula_wi → 'a) : 'a
  variant { size phi }
  ensures { result = k (nnfc phi) }
= ...

let nnfc_main (phi: formula_wi) : formula_wi
  ensures { forall v. eval_wi v phi = eval_wi v result }
  ensures { wf_negations_of_literals result }
= nnfc_cps phi (fun x → x)
```

C.2 CNFC

```
let rec cnfc_cps (phi: formula_wi) (k: formula_wi → 'a) : 'a
  requires{ wf_negations_of_literals phi }
  variant { phi }
  ensures{ result = k (cnfc phi)}
= ...

let cnfc_main (phi: formula_wi) : formula_wi
  requires{ wf_negations_of_literals phi }
  ensures{ forall v. eval_wi v phi = eval_wi v result }
  ensures{ wf_negations_of_literals result }
  ensures{ wf_conjunctions_of_disjunctions result}
= cnfc_cps phi (fun x → x)
```

D Defunctionalized Version

D.1 Types

```

type impl_kont =
  | KImpl_Id
  | KImpl_Neg impl_kont formula
  | KImpl_OrLeft formula impl_kont
  | KImpl_OrRight impl_kont formula_wi
  | KImpl_AndLeft formula impl_kont
  | KImpl_AndRight impl_kont formula_wi
  | KImpl_ImplLeft formula impl_kont
  | KImpl_ImplRight impl_kont formula_wi

type nnfc_kont =
  | Knnfc_id
  | Knnfc_negneg nnfc_kont formula_wi
  | Knnfc_negandleft formula_wi nnfc_kont
  | Knnfc_negandright nnfc_kont formula_wi
  | Knnfc_negorleft formula_wi nnfc_kont
  | Knnfc_negorright nnfc_kont formula_wi
  | Knnfc_andleft formula_wi nnfc_kont
  | Knnfc_andright nnfc_kont formula_wi
  | Knnfc_orleft formula_wi nnfc_kont
  | Knnfc_orright nnfc_kont formula_wi

type distr_kont =
  | KDistr_Id
  | KDistr_Left formula_wi formula_wi distr_kont
  | KDistr_Right distr_kont formula_wi

type cnfc_kont =
  | KCnfc_Id
  | KCnfc_OrLeft formula_wi cnfc_kont
  | KCnfc_OrRight cnfc_kont formula_wi
  | KCnfc_AndLeft formula_wi cnfc_kont
  | KCnfc_AndRight cnfc_kont formula_wi

```

D.2 Impl_Free Function

```

let rec impl_free_desf_cps (phi: formula) (k: impl_kont) : formula_wi
= match phi with
  | FNeg phi1 → impl_free_desf_cps phi1 (KImpl_Neg k phi1)
  | FOr phi1 phi2 → impl_free_desf_cps phi1 (KImpl_OrLeft phi2 k)
  | FAnd phi1 phi2 → impl_free_desf_cps phi1 (KImpl_AndLeft phi2 k)
  | FImpl phi1 phi2 → impl_free_desf_cps phi1 (KImpl_ImplLeft phi2 k)
  | FConst phi → impl_apply (FConst_wi phi) k
  | FVar phi → impl_apply (FVar_wi phi) k
end

with impl_apply (phi: formula_wi) (k: impl_kont) : formula_wi
= match k with
  | KImpl_Id → phi

```

```

| KImpl_Neg k phi1 → impl_apply (FNeg_wi phi) k
| KImpl_OrLeft phi1 k → impl_free_desf_cps phi1 (KImpl_OrRight k phi)
| KImpl_OrRight k phi2 → impl_apply (FOr_wi phi2 phi) k
| KImpl_AndLeft phi1 k → impl_free_desf_cps phi1 (KImpl_AndRight k phi)
| KImpl_AndRight k phi2 → impl_apply (FAnd_wi phi2 phi) k
| KImpl_ImplLeft phi1 k → impl_free_desf_cps phi1 (KImpl_ImplRight k phi)
| KImpl_ImplRight k phi2 → impl_apply (FOr_wi (FNeg_wi phi2) phi) k
end

let rec impl_desf_main (phi:formula) : formula_wi
= impl_free_desf_cps phi KImpl_Id

```

D.3 NNFC Function

```

let rec nnfc_desf_cps (phi: formula_wi) (k: nnfc_kont) : formula_wi
= match phi with
| FNeg_wi (FNeg_wi phi1) → nnfc_desf_cps phi1 (Knnfc_negneg k phi1)
| FNeg_wi (FAnd_wi phi1 phi2) → nnfc_desf_cps (FNeg_wi phi1) (Knnfc_negandleft
  phi2 k)
| FNeg_wi (FOr_wi phi1 phi2) → nnfc_desf_cps (FNeg_wi phi1) (Knnfc_negorleft
  phi2 k)
| FOr_wi phi1 phi2 → nnfc_desf_cps phi1 (Knnfc_orleft phi2 k)
| FAnd_wi phi1 phi2 → nnfc_desf_cps phi1 (Knnfc_andleft phi2 k)
| phi → nnfc_apply phi k
end

with nnfc_apply (phi: formula_wi) (k: nnfc_kont) : formula_wi
= match k with
| Knnfc_id → phi
| Knnfc_negneg k phi1 → nnfc_apply phi k
| Knnfc_negandleft phi1 k → nnfc_desf_cps (FNeg_wi phi1) (Knnfc_negandright k
  phi)
| Knnfc_negandright k phi2 → nnfc_apply (FOr_wi phi2 phi) k
| Knnfc_negorleft phi1 k → nnfc_desf_cps (FNeg_wi phi1) (Knnfc_negorright k phi)
| Knnfc_negorright k phi2 → nnfc_apply (FAnd_wi phi2 phi) k
| Knnfc_andleft phi1 k → nnfc_desf_cps phi1 (Knnfc_andright k phi)
| Knnfc_andright k phi2 → nnfc_apply (FAnd_wi phi2 phi) k
| Knnfc_orleft phi1 k → nnfc_desf_cps phi1 (Knnfc_orright k phi)
| Knnfc_orright k phi2 → nnfc_apply (FOr_wi phi2 phi) k
end

let nnfc_desf_main (phi: formula_wi) : formula_wi
= nnfc_desf_cps phi Knnfc_id

```

D.4 Distr Function

```

let rec distr_desf_cps (phi1 phi2: formula_wi) (k: wf_distr_kont) : formula_wi
= match phi1, phi2 with
| FAnd_wi phi11 phi12, phi2 →
  distr_desf_cps phi11 phi2 { distr_k = KDistr_Left phi12 phi2 k.distr_k }
| phi1, FAnd_wi phi21 phi22 →
  distr_desf_cps phi1 phi21 { distr_k = KDistr_Left phi1 phi22 k.distr_k }

```

```

| phi1,phi2 → distr_apply (FOr_wi phi1 phi2) k
end

with distr_apply (phi: formula_wi) (k: wf_distr_kont) : formula_wi
= match k.distr_k with
| KDistr_Id → phi
| KDistr_Left phi1 phi2 k →
  distr_desf_cps phi1 phi2 { distr_k = KDistr_Right k phi }
| KDistr_Right k phi1 →
  distr_apply (FAnd_wi phi1 phi) { distr_k = k }
end

let distr_desf_main (phi1 phi2: formula_wi) : formula_wi
= distr_desf_cps phi1 phi2 { distr_k = KDistr_Id }

```

D.5 CNFC Function

```

let rec cnfc_desf_cps (phi: formula_wi) (k: wf_cnfc_kont) : formula_wi
= match phi with
| FOr_wi phi1 phi2 →
  cnfc_desf_cps phi1 { cnfc_k = KCnfc_OrLeft phi2 k.cnfc_k }
| FAnd_wi phi1 phi2 →
  cnfc_desf_cps phi1 { cnfc_k = KCnfc_AndLeft phi2 k.cnfc_k }
| phi → cnfc_apply phi k
end

with cnfc_apply (phi: formula_wi) (k: wf_cnfc_kont) : formula_wi
= match k.cnfc_k with
| KCnfc_Id → phi
| KCnfc_OrLeft phi1 k →
  cnfc_desf_cps phi1 { cnfc_k = KCnfc_OrRight k phi }
| KCnfc_OrRight k phi2 →
  cnfc_apply (distr_desf_cps phi2 phi { distr_k = KDistr_Id })
  { cnfc_k = k }
| KCnfc_AndLeft phi1 k →
  cnfc_desf_cps phi1 { cnfc_k = KCnfc_AndRight k phi }
| KCnfc_AndRight k phi2 →
  cnfc_apply (FAnd_wi phi2 phi) { cnfc_k = k }
end

let cnfc_desf_main (phi: formula_wi) : formula_wi
= cnfc_desf_cps phi { cnfc_k = KCnfc_Id }

```

E Correction of defunctionalized functions

E.1 NNFC

```

predicate nnfc_post (k: nnfc_kont) (phi result: formula_wi)
= match k with
| Knnfc_id → let x = phi in x = result
| Knnfc_negneg k phi1 → let neg = phi in nnfc_post k phi result
| Knnfc_negandleft phi1 k → let hl = phi in nnfc_post k (FOr_wi phi (nnfc (
  FNeg_wi phi1))) result
| Knnfc_negandright k phi2 → let hr = phi in nnfc_post k (FOr_wi phi2 hr) result
| Knnfc_negorleft phi1 k → let hl = phi in nnfc_post k (FAnd_wi phi (nnfc (
  FNeg_wi phi1))) result
| Knnfc_negorright k phi2 → let hr = phi in nnfc_post k (FAnd_wi phi2 hr) result
| Knnfc_andleft phi1 k → let hl = phi in nnfc_post k (FAnd_wi phi (nnfc phi1))
  result
| Knnfc_andright k phi2 → let hr = phi in nnfc_post k (FAnd_wi phi2 hr) result
| Knnfc_orleft phi1 k → let hl = phi in nnfc_post k (FOr_wi phi (nnfc phi1))
  result
| Knnfc_orright k phi2 → let hr = phi in nnfc_post k (FOr_wi phi2 hr) result
end

let rec nnfc_desf_cps (phi: formula_wi) (k: nnfc_kont) : formula_wi
  ensures{ nnfc_post k (nnfc phi) result }
= ...

with nnfc_apply (phi: formula_wi) (k: nnfc_kont) : formula_wi
  ensures{ nnfc_post k phi result }
= ...

```

E.2 CFNC

```

predicate cnfc_post (k: cnfc_kont) (phi result: formula_wi)
= match k with
| KCnfc_Id → let x = phi in x = result
| KCnfc_OrLeft phi1 k → let hl = phi in cnfc_post k (distr hl (cnfc phi1))
  result
| KCnfc_OrRight k phi2 → let hr = phi in cnfc_post k (distr phi2 hr) result
| KCnfc_AndLeft phi1 k → let hl = phi in cnfc_post k (FAnd_wi phi (cnfc phi1))
  result
| KCnfc_AndRight k phi2 → let hr = phi in cnfc_post k (FAnd_wi phi2 hr) result
end

let rec cnfc_desf_cps (phi: formula_wi) (k: wf_cnfc_kont) : formula_wi
  requires{ wf_negations_of_literals phi }
  ensures{ cnfc_post k.cnfc_k (cnfc phi) result }
= ...

with cnfc_apply (phi: formula_wi) (k: wf_cnfc_kont) : formula_wi
  requires{ wf_negations_of_literals phi }
  requires { wf_conjunctions_of_disjunctions phi }
  ensures{ cnfc_post k.cnfc_k phi result }
= ...

```

E.3 Distr Defunctionalized Function Specification

```
predicate wf_distr_kont (phi: distr_kont)
= match phi with
| KDistr_Id → true
| KDistr_Left phi1 phi2 k →
    wf_negations_of_literals phi1 ∧ wf_conjunctions_of_disjunctions phi1 ∧
    wf_negations_of_literals phi2 ∧ wf_conjunctions_of_disjunctions phi2 ∧
    wf_distr_kont k
| KDistr_Right k phi →
    wf_negations_of_literals phi ∧ wf_conjunctions_of_disjunctions phi ∧
    wf_distr_kont k
end

type wf_distr_kont = {
  distr_k: distr_kont;
} invariant { wf_distr_kont distr_k }
by { distr_k = KDistr_Id }
```